Towards an Algebraic Specification of Quantum Networks

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ABSTRACT

The main attributes of quantum networks are the utilization of quantum phenomena, security guarantees, and availability of their main quantum resource – entanglement. The fundamental differences between classical and quantum information will require joint efforts in physics, engineering and computer science to make quantum networks functional and scalable. A common language must be established between the hardware and software community. We envision a foundational model for quantum network programming languages. Such a model should contain the essential constructs for programming quantum networks, allow for specification and verification of end-to-end entanglement distribution, and provide guidelines for composing network protocols.

KEYWORDS

quantum networks, entanglement, Kleene algebra

1 INTRODUCTION

Quantum networks are distributed systems providing communication services to distributed quantum applications, which bring numerous advantages over what is possible with classical applications. Most notable benefits are related to enhanced communication capabilities leading to increased security, examples being unconditionally secure client-server communication, blind cloud computing, and secure multi-party computation [11, 23, 33]. Distribution is also essential to expanding quantum computation beyond capabilities of individual quantum enabled computers to quantum clusters [17].

The basic unit of communication between two nodes in a quantum network is a distributed *Bell pair* or *EPR pair*[1] – a pair of quantum bits (qubits), one at each node, that are *entangled*. Entangled qubits are correlated in a much stronger way than can be achieved with classical information. As entanglement is a fundamentally quantum property, quantum networks must operate within the constraints of quantum hardware, one of which is *decoherence* – quick degradation of quantum state quality over time. Decoherence and other factors introducing noise and loss represent major obstacles to realizing long-distance quantum communication in the spirit of store-and-forward as in classical networks. All these factors turn end-to-end distribution of Bell pairs, which is the core quantum network service, into a *stateful* and *distributed* task that requires a lot of *runtime coordination*. Moreover it includes steps (e.g., distillation or initial entanglement generation) that have intrinsically *high probability of failure*.

The need for distributed coordination, statefulness, and failure-prone primitive operations all contribute to the complex behavior of quantum network protocols – distributed programs that govern end-to-end distribution of Bell pairs among remote nodes [12, 18]. The scarcity of resources in quantum networks (e.g., memory and communication qubits) prompts intensive resource sharing among quantum network protocols executing in parallel, exacerbating complexity even further. That same resource scarcity and parallel operation calls for *formal reasoning* about the network's behavior, enabling protocol optimization, efficient compilation to hardware, and safe co-existence of multiple protocols, in addition to verifying the correctness properties of individual protocols (e.g., that the Bell pairs are indeed being generated among the right nodes). Quantum networks already require tight coordination and are thus a natural fit for logically centralized architectures, similar to software-defined networking (SDN), allowing reasoning about global protocol behavior.

Our goal is to develop the formalism necessary to cater to global behavior analysis. To that end, we take inspiration from *NetKAT* [1] and outline our vision for a language and logic that can be used, respectively, for specification and reasoning about quantum network protocols. Such a language can become a foundation for a unified high-level interface between control and data plane in quantum networks, similar to what *OpenFlow* [19] and later *P4*/*P4-Runtime* [3, 10] became for classical networks.

2 BACKGROUND

Quantum networks are governed by the laws of quantum mechanics, which on one hand impose constraints on their design and on the other hand offer fundamentally new capabilities that are inherently impossible when only using classical information. The *no-cloning theorem* prevents copying unknown quantum states without irreversibly altering them. Thus, it is not possible to forward quantum information by the receive-copy-retransmit mechanism used in classical switches. However, the no-cloning theorem makes quantum communication unconditionally secure, leading to novel applications resistant to eavesdropping and man-in-the-middle attacks [23]. This way, even quantum networks with very modest resources can outperform classical communication.

[1] Named after Bell [2] and Einstein, Podolsky, Rosen[8].

The present work focuses on the core service provided by quantum networks, namely generation and distribution of entangled quantum states. Bell states form the basis of communication in quantum networks, since all distributed quantum applications can be built on top of (distributed) Bell pairs [4, 18]. (In particular, by fusion it is possible to obtain any multipartite state [12].) Bell pairs are maximally entangled quantum states, having the strongest possible quantum correlations among two-qubits states, which makes them easier to create, distribute, and apply error handling to.

In the following, we provide a high-level overview of key components in a quantum network. **End nodes** are devices running quantum applications, they must be capable of receiving and processing entangled pairs of qubits. Most physical architectures use a dedicated subset of qubits, called *communication qubits*, to generate distributed entanglement, and once a Bell pair is generated, the constituent qubits can be transferred into memory. A Bell pair is first generated locally by a quantum source, and then one or both of the entangled qubits are transmitted across the link(s) through *quantum channels*. However, the probability that a photon representing a qubit reaches the target node by direct transmission decreases exponentially with the distance. Entanglement distribution over long distances is implemented using **quantum repeaters**, making them the core active building block of quantum networks [4, 29]. A quantum repeater acts as an intermediate node between two end nodes, consuming the Bell pairs it shares with each node in order to create a new Bell pair connecting the end nodes. This physical process is known as *entanglement swapping*, and it can be extended with multiple quantum repeaters acting as intermediate nodes, as shown in the example in Figure 1. **Entanglement distillation** or purification is a process of generating a single Bell state from two or multiple imperfect entangled states. When distillation succeeds, the confidence in the state is improved. But distillation is inherently probabilistic, thus it substantially increases the resource demands [24]. In order to distinguish between successful attempts and failures, *heralded* entanglement generation schemes are deployed that announce successful generation. Future generations of quantum networks will likely deploy more sophisticated error and loss management strategies provided by more advanced technologies [11, 34]. **Classical channels** are another crucial component of quantum networks, as entanglement generation schemes depend on tight synchronization and signaling among a number of remote network entities.

3 PROBLEM STATEMENT

We follow the principles of a quantum internet outlined by the Internet Research Task Force's Quantum Internet Research Group [18]. This section describes our network model for end-to-end Bell pair creation, which we base on a thought experiment inspired by classical networks [18, §7].

In a quantum network, the control plane will manage routing and signaling (traffic exchanged over a classical channel), whereas the data plane will oversee generation of Bell pairs (qubits exchanged over a quantum channel). Several authors [17, 26] propose to embed quantum networks within classical networks and use the existing infrastructure to send and receive control messages. This may be achieved by adding a quantum data plane to the classical data plane in routers, and use classical and quantum links (both links are physical) to connect quantum capable end nodes. End-to-end Bell pair distribution between remote nodes is a stateful, distributed task that requires a lot of prior coordination. At the start, requests arrive to start creating Bell pairs between end points, indicating quality of service parameters. Each pair of end-points needs to create a *quantum virtual circuit* [12, 18], which entails identifying established paths between the endpoints. A routing algorithm then (with the use of a traffic engineering function, taking into account the capacity of the routers and channels and the resources already consumed by other virtual circuits) computes the optimal path, i.e., the best sequence of routers and channels that guarantee the requested quality of service [21, 27, 35–37]. Finally, signaling is used to specify the "forwarding rules" into the data plane of each quantum router on the path. (In figure Figure 1 the path between nodes A and C is the sequence of 0-links drawn in black, and the red virtual links depict the forwarding rules.)

This work focuses on the verification of these forwarding rules. In order to specify forwarding rules, quantum networks need sensible abstractions of the hardware, as in classical networks. We propose the following abstract building blocks for specifying the forwarding rules: `create` a Bell pair at a source, `transmit` a Bell pair over a short quantum link, `swap` Bell pairs via repeaters, and `distill` Bell pairs.

It is natural to ask whether we can benefit from drawing further analogies with popular approaches in classical networking. We were inspired by the success of NetKAT [1], a high level programming language and logic for specifying and reasoning about packet-switched networks, a part of the revolution following the rise of SDN. Our building blocks resemble NetKAT actions, where

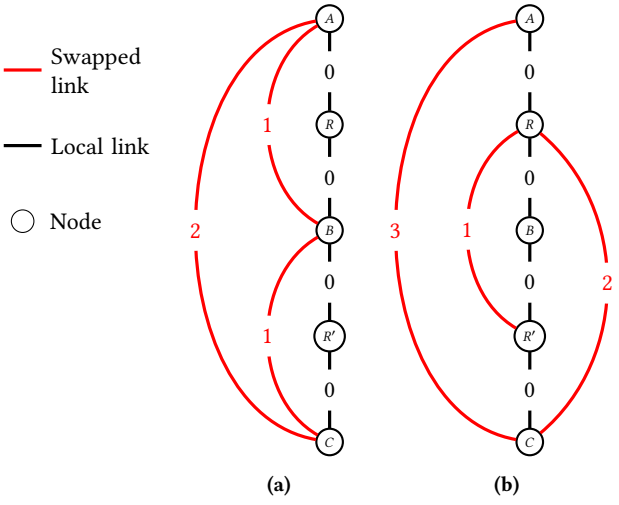

Figure 1: Two forwarding protocols on a 5-node network, generating an EPR pair between nodes A and C. Both protocols initially create Bell pairs at nodes A, B, and C, and transmit one qubit of the pair via a quantum link to a neighbor. Protocol (a) first performs swaps at R and R', and in the second round swaps A~B and B~C at B. By contrast, protocol (b) has three rounds of swapping, consecutively at B, R', and R.

assignments and tests are abstractions for packet field modifications and filters, respectively. Beside NetKAT actions, on which our *quantum actions* are based, we borrow another page from NetKAT and develop the notion of *quantum packets*, which represent Bell pair states being distributed through the network. These primitives, together with nondeterministic choice, sequential and parallel operators, and Kleene star, bring us closer to designing a quantum network programming language. The aim is to describe the language as an instance of *Kleene algebra*(or a related algebraic structure) and equip it with sound and complete equational theory, as it is done in NetKAT. The resulting language could be used for both programming and reasoning about quantum networks.

4 LESSONS FROM KLEENE ALGEBRAS

Kleene algebras (KAs) have been used for decades as algebraic structures of finite automata and regular events [14]. Kleene algebra with tests (KAT) is an extension of KA with Boolean actions that increase its expressiveness – it is known that KAT subsumes propositional Hoare logic [15, 16]. The only algebraic reasoning about quantum programs via KA that we know of was developed by Peng *et al.* [22], but they did not address a distributed setting. NetKAT language for classical networks is an instance of KAT whose equational theory is sound and complete with respect to its denotational semantics of nondeterministic functions over packets.

4.1 From Network Model to Language

Abstractly, a quantum network can be modeled as an automaton that coordinates the distribution of entangled qubits across the end nodes, along both physical and virtual quantum links. In this section, we faithfully bridge this abstraction with the network model described in Section 3.

We divide any given forwarding protocol into *rounds* representing time windows. Rounds contain atomic actions, which are executed in parallel. Sequential composition represents the progression from one round to the next one, and iteration is encoded using Kleene star. Atomic actions of a single round must all act on the set of Bell pairs available in the network in the corresponding time window, with race conditions emerging if resources are insufficient and there are not enough Bell pairs. In order for an individual atomic action to be successfully executed, it must first acquire a specific set of Bell pairs, said to be *required* by that action, from those available in the corresponding round and then successfully use these Bell pairs to generate a new entangled pair. If the required set of Bell pairs is not present in the network, then the action is not executed, and no Bell pair gets acquired, leaving these available to other actions of the same round. If the action acquires a set of Bell pairs but fails to successfully generate a new pair, the acquired Bell pairs are destroyed. A classical signal is sent from the quantum data plane to acknowledge the success or failure of each action. The next round of atomic actions then proceeds in the same manner on the set of Bell pairs produced or not consumed by the prior round.

It is not trivial to extend NetKAT language to its quantum counterpart since NetKAT was designed to model classical networks, which are quite different from quantum networks, as explained in Section 2. We now address some of the challenges related to modeling quantum forwarding protocols with an algebraic approach.

Bell pairs and network state. Bell pairs are the fundamental unit in quantum networks, like packets are in classical networks. Unlike packets, qubits carry no headers. Thus, control information needs to be sent via separate classical channels, which the nodes then correlate with the qubits stored in their memory. Another difference is that a Bell state consists of two qubits distributed across two nodes, and the two nodes must coordinate to ensure they are operating on qubits that belong to the same Bell pair. The identities of entangled nodes should be properly shared through the network, and the simplest way is to share their locations (we write $A{\sim}B$ for a Bell pair between nodes A and B). Locations are dynamically changed with each atomic action (at runtime), making the action stateful as opposed to the classical stateless packet switching.

In order to run a quantum network, it will be necessary to monitor it. To this end it may be convenient to define the network state, i.e., a partial function that assigns to each pair-location in the domain the number of Bell pairs at the corresponding end nodes. The total state would then represent the multiset of Bell pairs in the network at a given time. The notion of states is closely related to observations in concurrent KAs [13, 31]. (Observations are tests in a concurrent setting, making them more usable for verification).

Actions. An atomic action can be represented as an instance of a general form $I \triangleright_p O$, which consumes the multiset of required Bell pairs I and outputs the created Bell pair O with probability p. For example, the representation of a perfect swap of $A{\sim}B$ and $B{\sim}C$ at node B (denoted $\mathsf{sw}\langle A{\sim}C \mathbin{@} B\rangle$) is then $\{ A{\sim}B, B{\sim}C \} \triangleright A{\sim}C$, and a local create at node A (denoted $\mathsf{cr}\langle A\rangle$) can be represented as $\{\ \} \triangleright A{\sim}A$. Modeling failures of actions is necessary to capture decoherence and loss or to capture operations that may be inherently probabilistic (e.g., distillation returns $A{\sim}B$ or $\emptyset$) – we model these with random choice operation $\oplus_p$ inspired by [9, 28]. We remark that our actions also abstract the necessary control operations over the classical network. For example, Bell state measurement performed in the repeater during entanglement swapping requires two bits of classical control signals to be exchanged.

Policies. NetKAT policies encode the network topology and routing tables configured by a routing protocol. Denotationally, they are modelled as functions from packets to sets of packets. Policies are built from atomic policies encoding atomic actions, together with the nondeterministic choice and sequential composition combinators. Likewise, we consider whether we could model a quantum policy as an expression whose meaning encodes a forwarding protocol for Bell pair generation. For example, the forwarding protocol in Figure 1 (a) may be expressed as follows, where $\mathsf{tr}\langle A \to A{\sim}R\rangle$ represents physically forwarding half of the bell pair $A{\sim}A$ to node R and cr (create) and sw (swap) are as above:

$$\big(\mathsf{cr}\langle A\rangle \parallel \mathsf{cr}\langle B\rangle \parallel \mathsf{cr}\langle B\rangle \parallel \mathsf{cr}\langle C\rangle\big);$$
$$\big(\mathsf{tr}\langle A \to A{\sim}R\rangle \parallel \mathsf{tr}\langle B \to B{\sim}R\rangle \parallel \mathsf{tr}\langle B \to B{\sim}R'\rangle \parallel \mathsf{tr}\langle C \to C{\sim}R'\rangle\big);$$
$$\big(\mathsf{sw}\langle A{\sim}B \mathbin{@} R\rangle \parallel \mathsf{sw}\langle B{\sim}C \mathbin{@} R'\rangle\big);\ \mathsf{sw}\langle A{\sim}C \mathbin{@} B\rangle$$

Similarly, for the protocol (b):

$$\big(\mathsf{cr}\langle A\rangle \parallel \mathsf{cr}\langle B\rangle \parallel \mathsf{cr}\langle B\rangle \parallel \mathsf{cr}\langle C\rangle\big);$$
$$\big(\mathsf{tr}\langle A \to A{\sim}R\rangle \parallel \mathsf{tr}\langle B \to B{\sim}R\rangle \parallel \mathsf{tr}\langle B \to B{\sim}R'\rangle \parallel \mathsf{tr}\langle C \to C{\sim}R'\rangle\big);$$
$$\mathsf{sw}\langle R{\sim}R' \mathbin{@} B\rangle;\ \mathsf{sw}\langle C{\sim}R \mathbin{@} R'\rangle;\ \mathsf{sw}\langle A{\sim}C \mathbin{@} R\rangle$$

Quantum policies should be able to convey concurrent behaviours within one round, therefore Concurrent NetKAT [32] may be a useful starting point. In the execution of a policy, multiple nodes may simultaneously compete for the same Bell pairs. Potentially, the structure of a synchronous Kleene algebra [25] could handle the subset of actions that can be run in parallel. Furthermore, algebraic constructs like slices [1] and boxes [5] may facilitate modular construction of policies. Compositionality of algebraic structures fits well with the need for scalable and robust network architectures.

Histories. In NetKAT, histories convey paths or directions in which information (packets) travel from source to destination. This corresponds with the histories in Figure 2, which combine multiple undirected entangled links to create one end-to-end Bell pair. "Quantum histories" should record the behaviors that the forwarding rules produce. More concretely, they should capture the order of operations in a given execution of the policy. We remark that in our model, a single round could contain any atomic action, unlike the protocol stack of Van Meter et al. [20, 30] where each layer controls one type of operation.

Kleene star. Because of the probabilistic nature of operations, early generations of quantum networks will inevitably employ the strategy of repeated attempts of distillation and creation [20, 30]. As demonstrated by Pompili *et al.* [24], a pair of directly connected quantum nodes will repeatedly attempt to generate an entangled pair, until the heralding signal announcing success is received. In KA semantics, iterations are encoded with the Kleene star operator. We conclude this section with a challenge: Can we encode repeated attempts at creating Bell pairs using the Kleene star?

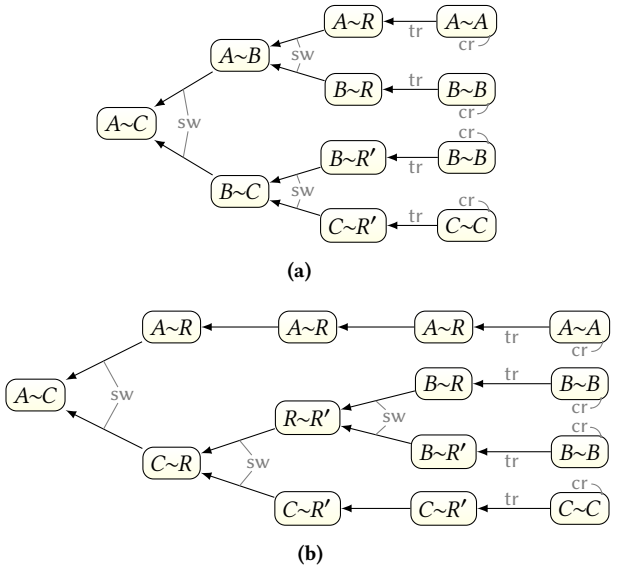

Figure 2: Histories of the two forwarding protocols in Figure 1, generating Bell pair $A{\sim}C$ along the same path. The (a) history has four rounds, two being swaps, and the (b) history has five rounds, three being swaps. Atomic actions are hinted in gray (not part of history).

4.2 Quantum Network Verification

The limitations of current hardware, such as low rates of Bell pair generation, short memory lifetimes, and limited numbers of communication qubits, make competition for resources unavoidable. This competition is the main challenge in reasoning about quantum network properties.

Routing and forwarding protocols, introduced in Section 3, will be responsible for allocating resources to the nodes. The forwarding rules depict the order in which quantum operations are performed during the generation of end-to-end entanglement, as shown in Figure 1. These rules are fundamentally different from those used in classical networks for hop-by-hop packet delivery.

By utilizing NetKAT's perspective on network verification, we propose to verify quantum networks using an *equational* approach, where properties of the network are reduced to checking equality between algebraic expressions (policies), which can established by direct manipulation of those expressions via established axioms. With a sound and complete axiomatization, we obtain a unified framework for reasoning about the network (a graph of physical and virtual links), policies, and their properties.

The following properties translate naturally from NetKAT.

- *Reachability.* The simplest reachability property would verify whether the execution of a policy represented by a set of forwarding rules generates the requested entanglement between end nodes.
- *Waypoint Correctness.* We may wish to guarantee that a forwarding protocol always performs the swapping operation through a certain node.
- *Traffic (Protocol) Isolation.* Composition of policies may lead to undesired behaviors, such as emerging race conditions. Could we prove non-interference properties that ensure isolation between policy executions?
- *Compilation.* Establishing the correctness of the compilation process is a necessary final step for ensuring correct deployment.

The following properties, which do not have a clear counterpart in NetKAT or any classical network analogy, could be posed as resource constraint checking problems.

- *Resource Utilization.* What is the number of required memory locations and communication qubits? For how many rounds must Bell pairs wait in the memory?
- *Quality of Service.* Do the generated end-to-end Bell pairs have the required fidelity or capacity?
- *Compilation.* Can we minimize the number of costly accesses to the network global state?

From histories, it is possible to read whether an underlying protocol obeys the hardware constraints (e.g., the number of communication and memory qubits, as illustrated in Figure 2), and also suggest how to optimize resource allocation over rounds. It is worth noting that Bell pairs between the same two nodes are indistinguishable for most applications, which could lead to more efficient provisioning of resources. In addition, the information recorded in histories could shed some light on the order among communication channels, investigated in [6].

5 OUTLOOK

Successful integration of classical and quantum networks will provide novel solutions for secure communication tasks, pave the way to distributed quantum computing, and enable other large scale applications of quantum communication technologies. Significant research and engineering efforts are still required until quantum networks reach full functionality. Our work focuses on the specification of routing and forwarding functions, taking into account the distinctive features of entanglement as the main communication resource. We are aiming towards a foundational model for quantum network programming languages.

Future research avenues include (1) identifying a solid semantic foundation, such as KA, on which the envisioned language and logic would be based, (2) translating existing quantum network protocols to the envisioned language, (3) developing tool support for the practical specification of protocols and verification of interesting properties. A key goal of (1) is to make the language semantics sound and complete, in order to allow for equational reasoning. In addition, the underlying axioms should faithfully encode the network behavior. The purpose of (2) is to assess the expressiveness of the language and its suitability for real world scenarios. Finally, (3) is a necessary step to make the approach suitable for practitioners. Such new tools would provide capabilities that are complementary to existing simulators like NetSquid [7].

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